

INHOMOGENEOUS EXACT SOLUTIONS WITH VARYING COSMOLOGICAL TERM

LUIS P. CHIMENTO¹ and DIEGO PAVÓN²

¹*Departamento de Física, Universidad de Buenos Aires,
1428 Buenos Aires, Argentina
E-mail: chimento@df.uba.ar*

²*Departamento de Física, Universidad Autónoma de Barcelona,
E-08193 Bellaterra (Barcelona), Spain
E-mail: diego@ulises.uab.es*

We study the evolution of LTB Universe models possessing a varying cosmological term and a material fluid.

I. INTRODUCTION

There is at present an increasing feeling in the astrophysic community that the cosmological constant is not zero but should contribute substantially to the mass-energy of the Universe -see Weinberg *et al.* [1] and references therein. This may be so if the energy of the quantum vacuum spontaneously decayed into matter and radiation, hence reducing the cosmological term to a value compatible with astronomical constraints -see for instance Overduin *et al.* [2] and references therein. On the other hand, recently it has been pointed out that because of sources evolution it may well happen that the Universe is in reality inhomogeneous and describable by the Lemaître-Tolmann-Bondi (LTB) metric [3]. Further motivations conducive to use inhomogeneous metrics can be found in Krasinski [4].

II. METRIC AND MODELS

We consider a spatially flat LTB metric

$$ds^2 = -dt^2 + Y'^2 dr^2 + Y^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad (Y = Y(r, t)) \quad (1)$$

whose source is a perfect fluid, with equation of state $P = (\gamma - 1)\rho$, plus a varying cosmological term $\Lambda(t)$. The non-trivial Einstein equations are

$$\rho + \Lambda = \frac{1}{Y^2 Y'} (\dot{Y}^2 Y')', \quad (2)$$

$$P - \Lambda = -\frac{1}{Y^2 \dot{Y}} (\dot{Y}^2 Y)'. \quad (3)$$

$$\frac{\ddot{Y}}{Y} + \left(\frac{\dot{Y}}{Y} \right)^2 - \frac{\ddot{Y}'}{Y'} - \frac{\dot{Y}}{Y} \frac{\dot{Y}'}{Y'} = 0 \quad (8\pi G = 1). \quad (4)$$

In general the solutions can be expressed as $Y(r, t) = R(r)^{2/3} Z(t)^{2/3\gamma}$. Next we summarize different scenarios of interest -see Chimento and Pavón [5] for details.

1. For γ and Λ constants one obtains

$$Y_1 = R^{2/3}(r) C_1^{2/3\gamma} \cosh^{2/3\gamma} \left(\frac{\sqrt{3\gamma\Lambda}}{2} t + \varphi_1 \right), \quad (5)$$

$$Y_2 = R^{2/3}(r) C_2^{2/3\gamma} \sinh^{2/3\gamma} \left(\frac{\sqrt{3\gamma\Lambda}}{2} t + \varphi_2 \right). \quad (6)$$

Obviously both sets of solutions have a final inflationary stage.

2. When $\gamma = \text{constant}$ and

$$\Lambda(t) = \frac{\lambda_0^2}{t^2} \quad (\lambda_0^2 = \text{constant}), \quad (7)$$

it follows that

$$Z(t) = C_1 t^{m_+} + C_2 t^{m_-} \quad (8)$$

where $m_{\pm} = (1 \pm \sqrt{1 + 3\gamma\lambda_0^2})/2$. Inflationary solutions may occur for large enough λ_0^2 .

3. For $\gamma = \text{constant}$ and

$$\Lambda = \lambda_0^2 t^{n-2} \quad (n \neq 0, 2), \quad (9)$$

the solution can be expressed as a combination of Bessel functions

$$Z = C_1 t^{1/2} J_{1/n} \left(\frac{\lambda_0}{n} \sqrt{-3\gamma} t^{n/2} \right) + C_2 t^{1/2} J_{-1/n} \left(\frac{\lambda_0}{n} \sqrt{-3\gamma} t^{n/2} \right). \quad (10)$$

The behavior at the asymptotic limits depends on n . For $0 < n < 2$ one has the following: (i) When $t \rightarrow 0$ one obtains $Z \sim C_1 t + C_2$ -one can choose $C_2 = 0$ to have the initial singularity at $t = 0$. (ii) When $t \rightarrow \infty$ there follows $Z \sim t^{\frac{1}{2}-\frac{n}{4}} \cos t^{n/2}$.

Likewise for $n < 0$: (i) when $t \rightarrow 0$ one obtains $Z \sim t^{\frac{1}{2}-\frac{n}{4}} \cos(t^{n/2} + \varphi)$. (ii) When $t \rightarrow \infty$ one obtains $Z \sim t$.

4. For $\gamma = \text{constant}$ and

$$\Lambda(t) = \lambda_0^2 + ce^{-\alpha t} \quad (c < 0), \quad (11)$$

where λ_0^2 , α and c are constants, again the general solution is a combination of Bessel functions

$$\begin{aligned} Z = & C_1 J_{\frac{\lambda_0}{\alpha}\sqrt{3\gamma}} \left(\frac{\sqrt{-3\gamma c}}{\alpha} e^{\frac{-\alpha}{2}t} \right) \\ & + C_2 J_{-\frac{\lambda_0}{\alpha}\sqrt{3\gamma}} \left(\frac{\sqrt{-3\gamma c}}{\alpha} e^{\frac{-\alpha}{2}t} \right), \end{aligned} \quad (12)$$

with

$$C_2 = - \frac{J_{\frac{\lambda_0}{\alpha}\sqrt{3\gamma}} \left(\frac{\sqrt{-3\gamma c}}{\alpha} \right)}{J_{-\frac{\lambda_0}{\alpha}\sqrt{3\gamma}} \left(\frac{\sqrt{-3\gamma c}}{\alpha} \right)} C_1 \quad (13)$$

in order to fix the initial singularity at $t = 0$. When $t \rightarrow 0$ one has $Z \sim t$. At the final stage, when $t \rightarrow \infty$ and $\Lambda \rightarrow \lambda_0^2$, one obtains the following asymptotic behavior

$$Y \approx R^{2/3}(r) e^{\frac{\lambda_0}{\sqrt{-3\gamma c}} t}. \quad (14)$$

For the particular case $\lambda_0^2 = 0$ and in the limit $t \rightarrow \infty$, there is a solution whose final behavior is

$$Y \approx R^{2/3}(r) t^{2/3\gamma}. \quad (15)$$

5. For $\gamma = \gamma(t)$ and $\Lambda = \Lambda(t)$ it can be found expressions for both quantities,

$$\Lambda(t) = \frac{4C^2(t-t_0)^{2n}}{3\gamma_0 n^2(n+1)^2} \left[1 + \frac{(t-t_0)^{n+1}}{C} \right]^{\frac{2-n}{n}}, \quad (16)$$

$$\gamma(t) = \gamma_0 \left[1 + \frac{(t-t_0)^{n+1}}{C} \right]^{-\frac{2+n}{n}}, \quad (17)$$

as well as an asymptotic solution for $Y(t, r)$

$$Y \approx R^{2/3}(r) T_0^{2/3\gamma_0} \left[\frac{(n+1)(n+2)}{n} (t-t_0) \right]^{2/3\gamma_0}, \quad (18)$$

where T_0, γ_0, t_0 , and C are constants. It is worthy of note that, for $t \gg t_0$ we have both $\gamma \rightarrow \gamma_0$ and $\Lambda \rightarrow 0$.

To examine the singular structure of the plane LTB metric (1) we have calculated the curvature scalar and evaluated it at the points where the coefficients of the metric Y'^2 and/or Y^2 vanish. All the solutions we have found for $\gamma = \text{constant}$ except (5) have a singularity at $t = 0$, i.e. the big-bang singularity.

III. CONCLUSIONS

We have found the coefficients of the LTB metric assuming that the early Universe possessed a time varying cosmological term, and that the adiabatic index of the material fluid were either constant or not.

- (a) All the solutions we have derived contain an arbitrary function of the radial coordinate.
- (b) For $\gamma = \text{constant}$ all the solutions, except (5) have a singularity at $t = 0$, i.e. the big-bang singularity.
- (c) Constant as well as varying cosmological terms give rise asymptotically to exponential inflation -see (5), (6) and (14).
- (d) For $\Lambda(t) \propto e^{-\alpha t}$ there exist solutions which behave as though the Universe were asymptotically matter dominated at late times when $\gamma = 1$, i.e. $Y \propto t^{2/3}$.

Acknowledgements

This work was partially supported by the Spanish Ministry of Education under Grant PB94-0718, and the University of Buenos Aires under Grant EX-260.

References

-
- [1] D.H. Weinberg, R.A.C Croft, Lars Hernquist, N. Katz and M. Pettini; *Report astro-ph/9810011*.
 - [2] J.M. Overduin and F.I. Cooperstock, *Phys. Rev. D* **58**, 43506 (1998).
 - [3] N. Mustapha, C. Hellaby and G.F.R. Ellis, *Mon. Not. R. Astron. Soc.* **292**, 817 (1997); gr-qc/9808079.
 - [4] A. Krasinski, *Inhomogeneous Cosmological Models* (Cambridge University Press, Cambridge, 1997).
 - [5] L.P. Chimento and D. Pavón, *Gen. Relativ. Grav.* **30**, 643 (1998).